Analytical Study of Finite/Infinite Impulse Response Optical Filter for Signal Processing Application

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Abstract

Re-circulating/infinite impulse response (feed-backward) and non-recirculating/finite impulse response (feed-forward) delay line are two structures that are basic to fiber-optic signal processing. Similarly, we can form the re-circulating and nonrecirculating system consists of many more coupler system. In this paper we have analyzed the response of each combination. Later on we try to connect the z-transform domain into Laplace by replacing $z = e^{sT}$. By replacing the z by the expansion of e^{sT} for some number of elements we can obtain the similar types of response to check the validity of our approximations. Here in s-domain we can easily apply control theory concepts on these transfer formulas. In the end we have tried to obtain the time domain response of filter by using the z-domain approximation in s-domain.

Keywords

Signal Processing Concept, Z-transform, S-domain, Optical Directional Coupler.

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Introduction

The low-loss and large bandwidth length product of optical fibers, together with advance in the manufacturing of fiber-optic components provide an attractive technology for processing broadband signal. Fiber-optic signal processing devices can constructed to perform various function, such as convolution, correlation, matrix operation, and frequency filtering, pulse-train generation, pulse compression, and matched filtering are also achievable[1]. Previous work has concentrated on classical tapped delay-line forms (transverse filters). Various mechanisms such as radiation due to bending and evanescent coupling by polishing the cladding down very close to fiber core have been used to realize the taps. In this paper attention is on different fiber optic structure, namely lattice (or ladder) forms, which can be used as alternatives for performing optical, signal processing[2]. In the two-coupler re-circulating delay line or infinite impulse response filter a single mode fiber loop is closed upon itself by two directional coupler, with the second coupler some portion of light back to the first coupler through the fiber loop. The two-coupler non-recirculating delay line or finite impulse response filter may also consist of two-directional coupler but in this case, the outputs of the first coupler are both fed forward to the second system.

Basic Fiber Structure: Elementary Concept

In the two-coupler re-circulating delay line a single mode fiber loop (with loop delay T) is closed upon itself by two directional coupler, with the second coupler feeding some portion of the light back to the first coupler through the fiber loop. Thus optical signal sent into the input port, say at x_1, circulate repeatedly around the loop, sending a portion of the recirculating intensity to the output port y_1 and y_2. The impulse response of the system consist of a series of decaying impulses equally spaced in time by the loop delay T, as shown in the Fig. 1 (a, c). A two coupler re-circulating delay line has already been used for frequency filtering.

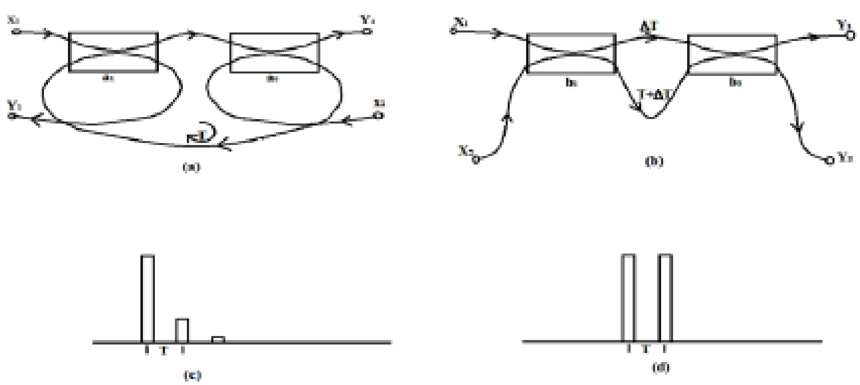

Figure 1. Basic fiber structure. (a) Two coupler re-circulating (feed-backward delay line). (b) Two-coupler non-recirculating (feed-forward) delay line. (c) Impulse response of the two-coupler recirculating delay line. (d) Impulse response of the two-coupler non-recirculating delay line.

The two-coupler nonrecirculating delay line also consist of two directional coupler, but in this case the output of the first coupler are both fed forward to the second coupler where they are recombined after a time delay T which is the time delay difference between the two feed-forward fiber lines. The impulse response of the system is composed of just two impulses spaced by the amount T in time as shown in Fig.1 (b, d). The input (E_1, E_2) output (E_3, E_4) relationship of the directional as shown in Fig. 2 can be described by a 2×2 complex transfer matrix[2].

$$\begin{pmatrix} E_3 \\ E_4 \end{pmatrix} = (\gamma)^{\frac{1}{2}} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \tag{1}$$

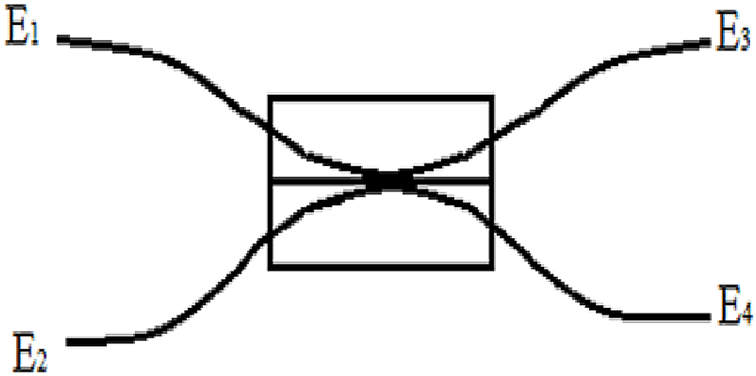

Figure 2. Schematic diagram of a fiber-optic directional coupler.

Hence, E_1, E_2, E_3, and E_4 are assumed, for simplicity, to have the same state of polarization. It should be mentioned that the coupler itself exhibits very little dependence on the state of the polarization of the input fields. For a symmetric coupler ($A = D$ and $B = C$) can be rewritten as

$$\begin{pmatrix} E_3 \\ E_4 \end{pmatrix} = (\gamma)^{\frac{1}{2}} \begin{pmatrix} \sqrt{1-\kappa} & j\sqrt{\kappa} \\ j\sqrt{\kappa} & \sqrt{1-\kappa} \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \tag{2}$$

where square root of κ is the amplitude coupling coefficient, and $j = \sqrt{-1}$ indicates a 90^o phase shift for the coupled signal. Furthermore, in the case when a temporarily incoherent light source is used, the input and output light intensities can be related to each other by the following expression;

$$\begin{pmatrix} I_3 \\ I_4 \end{pmatrix} = (\gamma) \begin{pmatrix} 1-\kappa & \kappa \\ \kappa & 1-\kappa \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \tag{3}$$

where (I_1, I_2) and (I_3, I_4) are the input and output intensities, respectively, γ is the overall intensity transmission factor, and κ is the intensity coupling coefficient[3]. These structures are the subject of our discussion in the next section.

Lattice Structure

In this section we introduce two fiber optic lattice configuration; re-circulating (feed-backward) and nonrecirculating forms shown an N^{th} order feed forward forms. Figure 3(a) shows an N^{th} order feed backward (re circulating) fiber optic lattice structure[4]. In this section, we are interested in this case where T is the same for all section, but this restriction would not be necessary in general. This structure is obtained by interconnecting N two coupler re circulating delay lines in tandem.

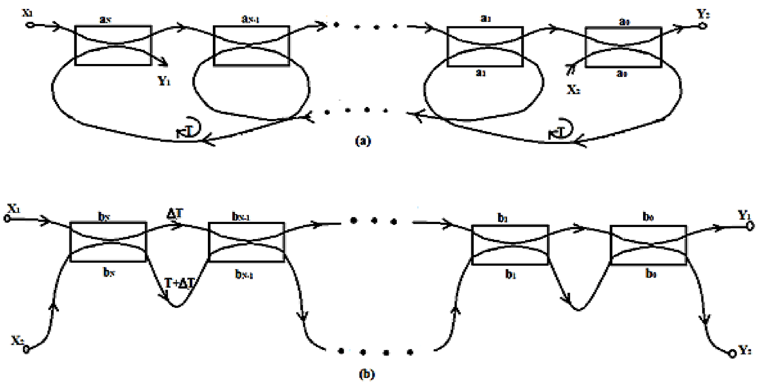

Figure 3. Schematic diagrams of the N^{th} order fiber optic lattice structure with $N + 1$ coupler and N delay elements (a) Feed backward configuration. (b) Feed forward configuration.

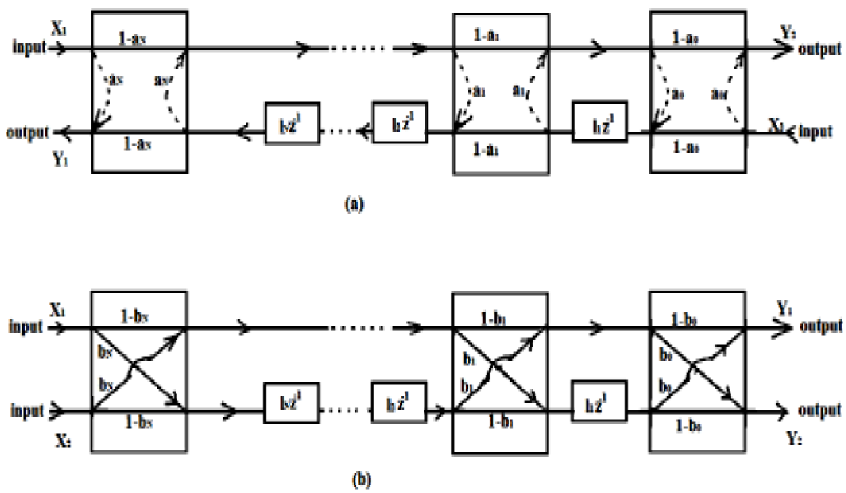

Figure 4. Equivalent diagrams of the N^{th} order fiber optic lattice structure with $N + 1$ couplers and N delay elements (a) Feed backward configuration. (b) Feed forward configuration. (I_1 is the intensity of the i^{th} section).

As seen in Fig. 3(a), this structure has one feed forward and one feed backward fiber line. In contrast, Fig. 3(b) illustrates an N^{th} order feed forward (nonrecirculating) fiber lattice structure which is implemented by cascading N two coupler non re circulaing delay lines. In this configuration, there are two feed forward lines with no feed backward line. Both of these lattice configurations have two ports with two terminals at each port (two inputs, two outputs)[5]. Figure 4 shows the equivalent diagram of the feed backward and feed forward lattice structure. These two lattice forms can also be combined to construct more sophisticated fiber optic structures.

Mathematical Analysis

In order to increase our ability to handle more complex system problems, we utilize transform techniques for describing the signal. We particularly use the Z-transform, later we convert Z-transform into Laplace domain[6]. The Z-transform, $F(z)$ of a signal, $f(k)$ is defined by the following expression

$$F(z) \triangleq \sum_{k=-\infty}^{\infty} f(k) \cdot z^{-k} \tag{4}$$

Where k is the time index, which is an integer multiple of the basic time delay ($k = nT$), and z is the transform variable which represents a unit time advance (z^{-1} represents a unit time delay)[7]. Also since we know that $s = j\omega$, hence $z = e^{sT}$. Hence we can easily connect Z-transform domain into s-domain. Here s-domain is Laplace transform and we can easily apply control theory concepts on these transfer formulas. This $e^{j\omega T}$ describes a unit circle centered at the origin of the Z plane. The unit circle also plays an important role in problems regarding stability. Figure 5, shows the z-plane with the unit circle[8].

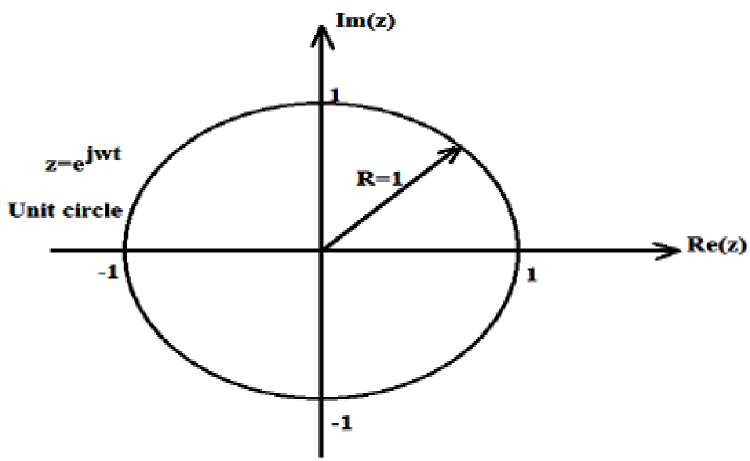

Figure 5. Z-plane with the unit circle.

To ensure stability, all the poles of the system transfer function must be inside the unit circle or in s domain, all the poles zeros should be in entire left half of s plane. Fiber lattice filters incorporating passive elements (no gain) are sure to be stable[9].

Two Coupler Re-circulating/Non-Recirculating Delay Lines

Figure 6 shows the two coupler re-circulating delay line, where X_1 and Y_1 are input and output respectively. The relation between input and output for two coupler re-circulating system is given by,

$$Y_1 = a_1 X_1 + (1 - a_1)^2 a_0 l_1 X_1 z^{-1} + (1 - a_1)(1 - a_0) l_1 z^{-1} X_2$$

when $X_2 = 0$;

$$H_{11}(z) = \frac{a_1 + (1 - 2a_1) a_0 \iota_1 z^{-1}}{1 - a_1 a_0 \iota_1 z^{-1}} \tag{5}$$

where a_1 and a_0 are the intensity coupling coefficient of the couplers, l_1 is the loop intensity transmittance of the system. As shows this system has one zero at

$$z = \frac{(2a_1 - 1)}{a_1} a_0 \iota_1 \tag{6}$$

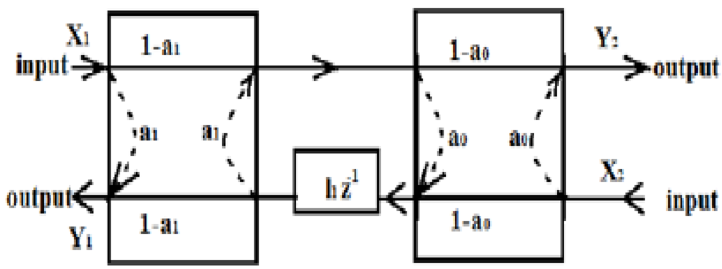

Figure 6. Two coupler re-circulating delay line.

Figure 7, shows the plot of normalized magnitude and phase response for this case. Two coupler re-circulating delay line, with X_1 as the input and Y_2 as the output as shown in Fig. 6. We can write the relation between the input X_1, X_2 and output Y_2 as follow

$$Y_2 = (1 - a_1)(1 - a_0) l_{11} X_1 + (1 - 2a_0) a_1 l_1 X_2 z^{-1} + a_0^2 a_1 l_1 X_2 z^{-1} \tag{7}$$

Putting $a_0 X_2 = Y_2$ in third term and $X_2 = 0$ in second term of above equation;

$$Y_2 = (1 - a_1)(1 - a_0) l_{11} X_1 + a_0 a_1 l_1 Y_2 z^{-1} \tag{8}$$

Hence;

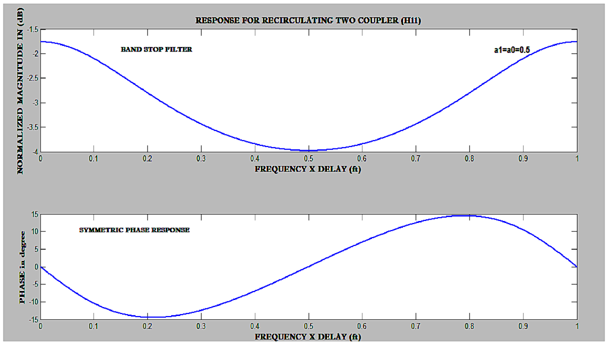

Figure 7. Plot of normalized magnitude and phase response for re-circulating two coupler systems (H_{11}).

$$H_{21}(z) = \frac{Y_2}{X_1} = \frac{(1-a_1)(1-a_0)\iota_{11}}{1 - a_1 a_0 \iota_1 z^{-1}} \tag{9}$$

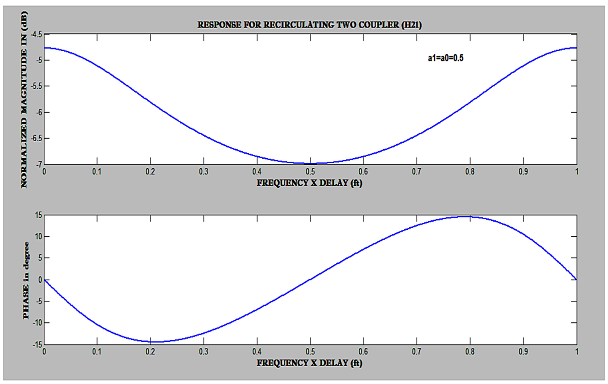

Figure 8. Plot of normalized magnitude and phase response for re circulating two coupler systems (H_{21}).

where l_{11} is the intensity transmission factor of the forward fiber line. In this case, the system has one zero at the origin and one pole at $z = a_1 a_0 \iota_1$. Figure 8, shows the plot of normalized magnitude and phase response for this case.

Two coupler non-recirculating delay line, as shown in Fig. 9, with X_1 as the input and Y_1 as the output. As we know that relation between the input and output in case of two coupler.

$$Y_1 = (1-b_1)(1-b_0)X_1 l_{11} + b_1 b_0 X_1 z^{-1} + (1-b_1)b_0 l_{12} z^{-1} X_2 + (1-b_0)b_1 l_{11} X_2$$

As X_2 is zero, thus

$$H_{11}(z) = \frac{Y_1}{X_1} = (1-b_1)(1-b_0)\iota_{11} + b_1 b_0 \iota_{12} z^{-1} \tag{10}$$

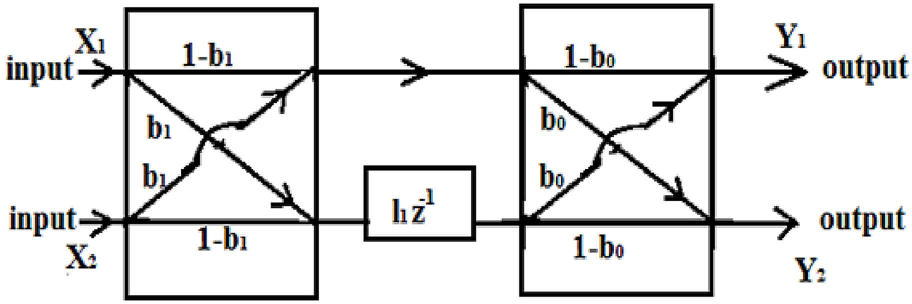

Figure 9. Two coupler Non-reciculating delay line.

where b_1 and b_0 are the intensity coupling coefficient of the couplers, and l_{11} and l_{12} are the intensity transmittances in the two feed forward fibers lines. This system has one pole at the origin and one zero at,

$$z = -\frac{b_1 b_0 \iota_{12}}{(1-b_1)(1-b_0)\iota_{11}}. \tag{11}$$

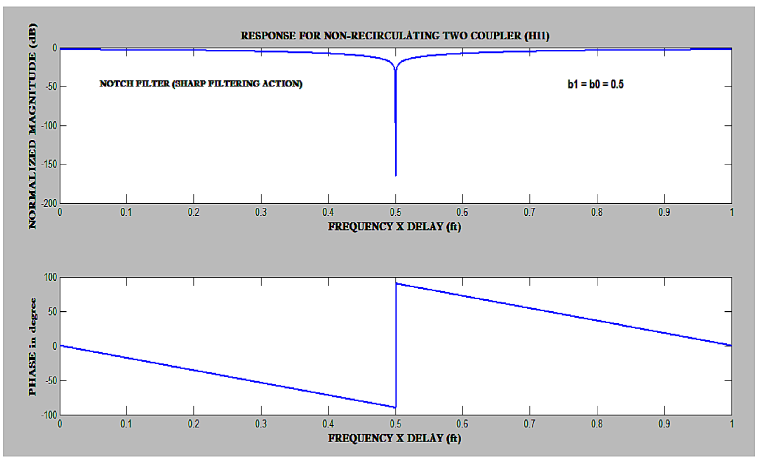

Figure 10. Plot of normalized magnitude and phase response for non re-circulating two coupler system (H_{11}).

Note that those zeros or poles which are located at the origin do not affect the frequency response of the system (except for a linear phase factor). For this reason, we refer to the two coupler re-circulating delay line, with Y_2 as the output, as a first order (one pole) all pole system. Figure 10, shows the plot of normalized magnitude and phase response for this case. The two coupler non-recirculating delay line, with Y_2 as the output, are referred to as first order (one zero) all zero and first order (one pole) pole zero system, respectively. The pole zero diagram of the system discussed in this section is illustrated in Fig. 11. This diagram contains the information about the frequency response of the corresponding system.

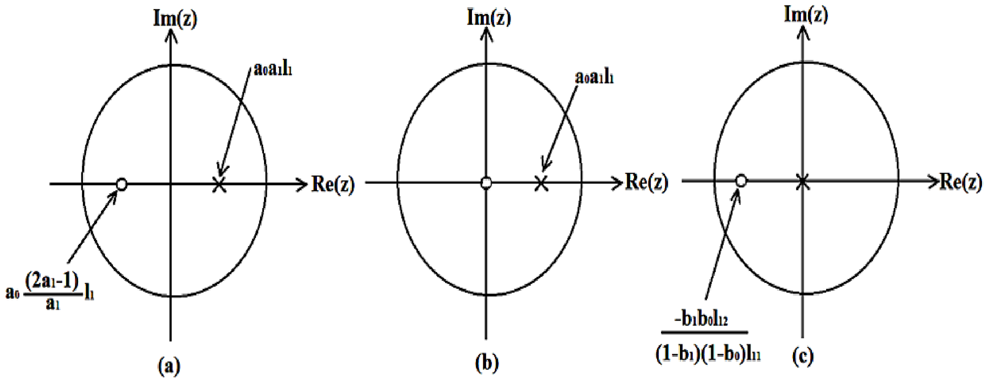

Figure 11. Pole zero diagrams of the two coupler delay lines with X_1 as the input. (a) Two coupler re circulating delay line, with Y_1 as the output. (b) Two coupler re circulating delay line with Y_2 as the output. (c) Two coupler non recirculating delay line with Y_1 as the output.

Conversion from Z-Domain to S-Domain

Since we know that $s = j\omega$, hence $z = e^{sT}$. Hence, we can easily connect z transform domain into s domain. Here s domain is Laplace transform and we can easily apply control theory concepts on these transfer formulas.

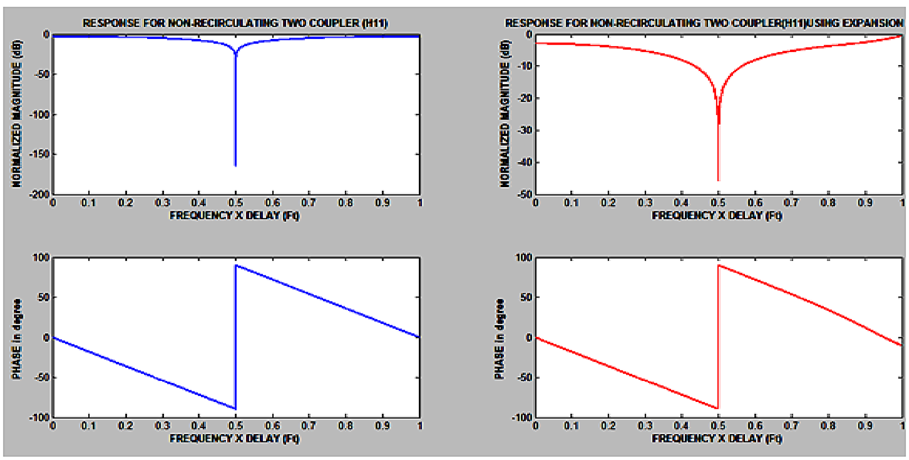

Figure 12. Comparison of response for non-recirculating two coupler system (z-domain and its approximation in s domain expansion).

This $e^{j\omega T}$ describes a unit circle centered at the origin of the z-plane. The unit circle also plays an important role in problems regarding stability. Hence, for two coupler system e^{sT} is approximated up to 13 numbers of elements.

$$z = e^{sT} = 1 + sT + \frac{(sT)^2}{\lfloor 2} + \frac{(sT)^3}{\lfloor 3} \ldots\ldots\ldots \tag{12}$$

Thus, we can compare the response for non-recirculating and re circulating system in z-domain and s-domain with approximation of expansion as shown in Fig. 12. Figure 12 shows the similarity between z-domain and s-domain using the approximate expansion of e^{sT}. Now, applying this approximation we can obtain the response of non-recirculating and re circulating system as follow. As we know from eq. (10) that,

$$H_{11} = (1 - b_1)(1 - b_0)\iota_{11} + b_1 b_0 \iota_{12} z^{-1} \tag{13}$$

By substituting the expansion of z-in terms of s-domain by using eq. (12) one can obtain the various response parameters in s-domain or continuous domain. We computed the plot of the normalized magnitude and phase response for the two coupler non-recirculating system using the expansion of $z = e^{sT}$. All the parameters represent the notch filtering action. The response is very much similar to that of the Fig. 10. For a re-circulating system the similar type of graph can be obtain using the expansion approximation. As we know that; in case of re circulating system H_{11} is given by; eq. (5), as

$$H_{11}(z) = \frac{a_1 + (1 - 2a_1)a_0\iota_1 z^{-1}}{1 - a_1 a_0 \iota_1 z^{-1}} \tag{14}$$

By using the eq. (12), we can replace the value of z-in terms of s; hence we obtain s-domain response of various parameters. We computed the plot of normalized magnitude and phase response of the various parameter of the two coupler re-circulating system using the approximation of the $z = e^{sT}$ up to 13 numbers of the elements. The response is found to be very much similar to that of Fig. 7 and Fig. 8. After verification of our result in s-domain with z-domain, we try to compute the time domain response of the systems. For non-recirculating system the time domain response can be computed by taking inverse Laplace transform of transfer function.

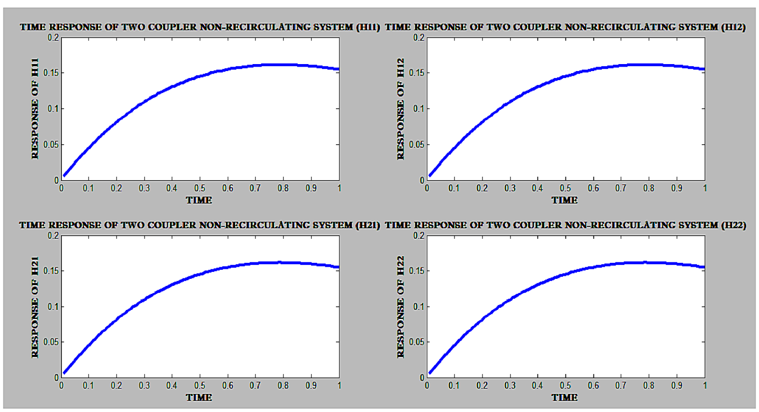

Figure 13. Time domain response of various parameter of two coupler non-recirculating system.

Figure 13 gives the idea about the time response of the various parameters of two coupler non-recirculating system. It is apparent that these waveforms are equivalent to that of the low pass filtering action. Similarly, for re-circulating system one can represents the plot of the time response of various parameters of the two coupler re-circulating system. Apparently these waveforms must be the characteristics of the high pass filter.

Conclusion

In this paper we have studied about the finite impulse response versus infinite impulse response optical filters. From this study one can easily compute the group delay and group velocity dispersion of various types of lattice structure. The results have been presented up to two coupler re-circulating/non-recirculating lattice structure. However, it can be easily extended to the higher order of the system. It is apparent that as we move from lower order system to higher order system the response of the lattice structure improves substantially. From our proposed study one can study the tunable optical ring resonator incorporating a symmetric/asymmetric Mach-Zehnder interferometer (MZI) and phase shifter. The optimal resonance state of the ring resonator with different geometries can be achieved by tanning the two embedded phase shifter. Design of the optical sideband filter is also proposed work to work out on the basis of this study.

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