Motion Artifact Removal Using Empirical Mode Decomposition Using Ks Test Blind Source Separation and Wavelet Transform

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Abstract

For medical science image processing, the elimination of artifacts from physiological signals is an essential step. The acuteness in performance of healthcare technology has upgraded from the current hospital centric environment towards a portable ubiquitous approaches. The enormous cost of fine equipment and ensemble technologies has created a window for mathematical models that approximates the signal dynamics towards a superlative performance. The uncertainty in subsequent performance of these approaches introduced a dedicated research and past few decades have witnessed considerable improvement. In this paper an enhanced empirical approach to model the artifacts of a free signal is described followed by filtering mechanism using ICA and DDWT. The input EEG is a single channel and is converted into multichannel for ICA operations. The multi-channel EEG constructed using EEMD is filtered with fast ICA algorithm and DDWT is employed to reject any traces of artifacts left in signal. This system is tested on various platforms of evaluation and results pronounce the eligibility of proposed algorithm to stand on top of currently deployed algorithms on account of significant improvement in results.

Keywords

EEG, EEMD-ICA, ICA, DDWT, EEMD-DDWICA

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Introduction

The design of a physiological signal is a long followed approach that illustrates the present condition about health of an individual. The dynamic research in medical sciences towards superlative health assessment calls for paramount accuracy with low computing cost in signal recordings and imaging. The life of an instrument to function with minimal operational and maintenance cost is one of primary factors that defines the probability of its selection as the practically acceptable technology. Also the simplicity of an instrument is directly measured as its capability to function in standard environment without failure in operation. Further, the complexity of an instrument holds a direct relationship with cost ensemble in it.

It is evident that measurement of physiological signals in even surgical environment is accustomed with some noise also referred as artifacts in medical terms. These artifacts are unwanted signals generated due to unregulated sources besides the source under consideration. The artifacts in neural signals have two prominent sources other than the machine and environment noise. The muscular and ocular activities of an individual generate electric pulses of low amplitude and frequency that falls in filter range of sensors and recording equipment. Artifacts rejection hence is a fundamental subject of research and is well researched[1]. This paper, considers the artifacts caused due to motion. Till this point numerous applications of ICA, wavelets and adaptive filters are proposed in same context of research[2].

Normally, a common approach is to reject all EEG epochs containing the signal amplitude larger than some selected value. These schemes are rigid and do not lend themselves to adaptation. Moreover, a portion of data will be lost. A component based automated separator of artifacts is required to overcome this issue based on linear decomposition of signals into source components. The components give individual nature of information, where artifacts information combines into separate sources and reconstruction of signals without this source are claimed as artifact free information. Hyvärinen[3] P. Levan et al.[4] on a 30s window size applied FastICA Algorithm using EEGLAB platform. They developed an automated system for artifact removal based on ICA and Bayesian Classification. Authors of[5] analyzed EEG waves by DWT for frequency domain analysis. Authors of[4] presented a statistical method based on wavelet transform to mimic ocular artifacts in EEG. Haslaile Abdullah[6] proposed wavelet based image processing technique at various window sizes. 1-D double density and 1-D double density complex were tested at window size of 10s, 30s, 60s and 300s for EEG signals. Mijovic et al[7] compared the EEMD-ICA algorithm against two other techniques capable of operating on single channel recordings, namely Single-Channel ICA (SCICA) and Wavelet-ICA (WICA). The algorithms were tested and compared on simulated data and the EEMD-ICA algorithm was then employed on real EEG and EMG data. The results show that the SCICA algorithm has the worst performance when comparing the relative root mean square error (RRMSE).The WICA algorithm, although comparable to the EEMDICA technique, showed a slightly weaker performance in the simulations.

The organization of paper is as follows: In this paper, the limitation of ICA of operations only on multichannel signals is overcome using EEMD approach. The EEMD decomposes a single channel EEG to a multi-channel function and sourced to fast ICA algorithm for filtering. The results of ICA were improved using DDWT algorithm as the output of ICA contains the traces of artifacts. The decomposition through DDWT was performed on frequency domain of signal obtained via frequency transformation (Fast-Fourier). The results brace the splendid performance of proposed architecture and paper ends with a conclusion.

EEMD

Empirical mode decomposition (EMD) is a method, first defined in 1998[8], for nonlinear signal processing and is well suited to non-stationary data. The method decomposes a time series signal into multiple "intrinsic mode functions" (IMFs). The EMD technique differs from other techniques, such as Wavelet analysis. In EMD the decomposition of the signal is a data driven process whereas Wavelet analysis relies on the selection of the appropriate wavelet. As the technique is data driven, it is therefore adaptive in nature, making it very flexible.

The IMFs are functions that satisfy two separate conditions:(1) over the full length of the data set the number of maxima and the number of zero crossings must be the same or differ at most by one and (2) at any point over the data set, the mean value of the envelope defined by the maxima and the envelope defined by the minima must be zero[8].

There are a number of steps taken to calculate the IMF of a given time series, $x \in R^N$ where N is the number of samples.EMD is implemented using a sifting process that uses only local extrema. Step 1 involves finding all the local maxima and minima over the full length of the time series. Next, the maxima are connected using a cubic spline creating an upper envelope, with the same process repeated for the local minima. Step 2 then involves calculating the average of the two envelopes m and subtracting this average from the data signal, resulting in a new signal $h = r_0 - m$, where $r_0 = x$.This signal h now becomes the new data signal and steps one and two are repeated until h complies with the properties of IMFs, detailed above. When this occurs, the current data set h becomes the first IMF (c_1). The above steps are then repeatedon the residual signal $r_1 = r_0 - c_1$. The sifting process stops when the residual signal r_n becomes a monotonic function.Once calculated, the IMFs c_j have the property that if they areadded together then the original data x (or r_0) is reconstructed, i.e.

$$x = \sum_{j=1}^{n} c_j + r_n \tag{1}$$

Where, r_n is the residual of data after n IMFs components are extracted[9].

The method of detecting IMFs is sensitive to the amalgam of undesired signal components present in surrounding. These noises affect the EMD process and mode mixing is used to overcome the disparate scale oscillations with amplitude in near range of the IMFs peaks and available randomly in whole dataset[10]. Also the momentary spectral components are sometimes misinterpreted as artifact components. An enhanced version of empirical mode decomposition[9] minimized this mode mixing quandary. Another version of EMD known as Ensemble-EMD (EEMD) employs the average value of EMD ensembles that filters out the IMFs from given signal. Every iteration of EMD process is identical and independent in nature towards the undesired signals.

The IMFs from the EMD process are filtered to discard the artifacts present in them (details available in following sections). The filtered IMFs are further processed for second stage filtering through wavelet transform for better outputs of evaluation parameters and signal quality.

Independent Component Analysis

ICA employs statistical and computational techniques for separating the mixture of signals into independent components. The IMF(s) generated by EEMD are further sampled to ICA as input in terms of $C = [c_1, c_2, c_j \ldots c_n]$ are generated by independent sources $S = [s_1, s_2, s_j \ldots s_n]$ where A is the $n \times m$ mixing matrix.

$$S = AC \tag{2}$$

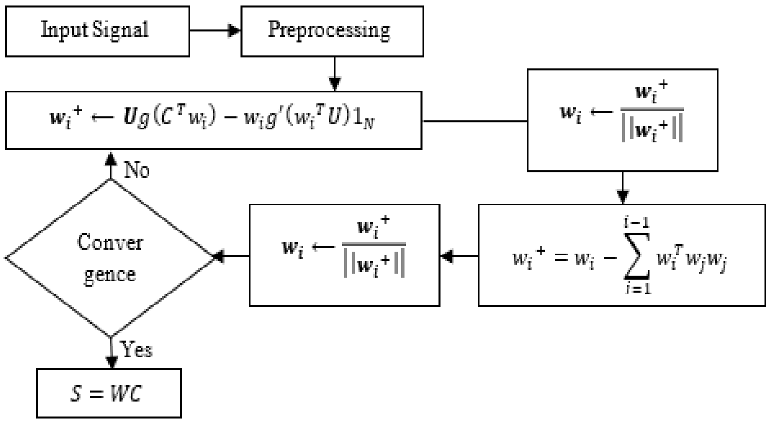

Figure 1. FastICA algorithm

Here, $\mathbf{w_i}$ is column vector and $w_i{}^+$ is temporary variable,$g(.)$ *and* $g^{'}(.)$ represents first and second derivate of nonlinear and non-quadratic functions. When the convergence is received w_{i+1} must be made orthogonal with equation c to differentiate the new components. In Z row those represent artifacts are set to be zero.

One sample Kolmogorov-Smirnov test[11] measures the non-gaussianity in ICA. The output of ICA based on gaussianity are classified in two types of signals i.e. signals either having random behavior or signals with stationary waveforms. The KS test separates both these signals by allocating a '0' and '1' to these signals respectively. The independent source components being artifacts are selected and relative matrix C column components are defined as null.

The KSTEST is implemented as the function in MATLAB and the equation against it is:

$$D^* = \max_C \left(\left| \widehat{F}(C) - G(C) \right| \right) \tag{3}$$

Where, $\widehat{F}(x)$ is empirical CDF and $G(x)$ is the CDF of hypothesized distribution. The original source signal is thus:

$$S' = (W) * C' \tag{4}$$

The FastICA algorithm is a fixed point iterative method that provides the highest value of non-Gaussianity to evaluate the statistical independence. The FastICA algorithm can also be viewed as approximative application Newton approximation. This algorithm looks for the course of weight vector by elevating the value of non-Gaussianity of projection $w_i^T x$ for input data x (figure 1). When the computation of the weight parameters completes, the estimated independent components will be generated by applying the unmixing matrix to EEG source signals.

With this technique spectral enhancement can be achieved but at the same time it very difficult to estimate the variances of Independent components[12]. To make more precise analysis in time and frequency further wavelet technique is adopted[4].

Discrete Wavelet Transform

The wavelet technique came into existence to overcome the resolution limitations of spectral analysis of Fourier Transform and denoise the corrupted signal. Selesnick and Ivan W[13] presented the Double Density Discrete Wavelet Transform as the custom-design form of Daubechies orthonormal wavelet transform by manipulating the window size of filter followed by entertaining certain critical polynomial aspects in an oversampled framework. The sensing time of DDWT is twice as that of the conventional DWT due to number of samples at each step. This algorithm was further customized as 1-D, 2-D up to Double Density complex[14]. The input signals are in space-time domain and reflects discreet properties because of oversample analysis and respective filterbank synthesized (figure 2).

Figure 1 depicts the frequency response of filters $h^0,\ h^1,\ h^2$ for analysis-bank filters of the defined size 7, 7, 5 respectively. The first is a low pass and remaining two are high pass filters in nature in constraints of congruent response of frequency magnitude. Iteration is employed for generation of oversampled filter bank to measure the h^0 response of both synthesis and filter banks. An application of DD-DWT was illustrated by Selesnick et al.[13] that analyses the unity scaling function $\emptyset$ and multi resolution along with φ^1, φ^2 wavelets (see figure 1). A "half-delay" property $\varphi^1\ (t) = \varphi^2\ \left(t - \frac{1}{2}\right)$ can be satisfied by application of wavelets.

The double density DWT is applying the oversampled filter bank on a low-pass sub band signal $c(n)$. In this paper the double density algorithm is used along with the filter banks for sampling purpose.

Figure 2 represents the formation of DD-DWT along with FIR filters through an oversampled filter bank. The filters employed are the low and high pass $h^0,\ h^1,\ h^2$ respectively as discussed above.

To develop the perfect reconstruction condition we use standard multirate identities to write $Y(z)$ in terms of $X(z)$

$$Y(z) = \frac{1}{2}\left[H_0(z)H_0\left(\frac{1}{z}\right) + H_1(z)H_1\left(\frac{1}{z}\right) + H_2(z)H_2\left(\frac{1}{z}\right)\right] S^{'}(z) \tag{5}$$

$$= \frac{1}{2}\left[H_0(z)H_0\left(-\frac{1}{z}\right) + H_1(z)H_1\left(-\frac{1}{z}\right) + H_2(z)H_2\left(-\frac{1}{z}\right)\right] S^{'}(-z) \tag{6}$$

The filter bank employed to structure the double-density discreet wavelet transform resembles the wavelet frame with $\phi(t)$ as scaling function and $\psi_1(t)$ and $\psi_2(t)$ wavelets. According to the documents of dyadic wavelet space

$$V_j = \underset{n\epsilon Z}{Span}\left\{\phi(2^j t - n)\right\} \tag{7}$$

$$W_{i,j} = \underset{n\epsilon Z}{Span}\left\{\psi_i(2^j t - n)\right\}, \quad i = 1, 2 \ldots n. \tag{8}$$

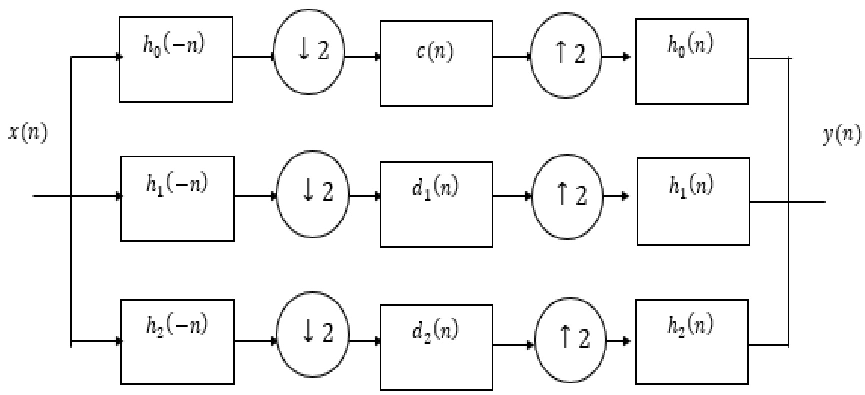

Figure 2. frequency responses for the analysis-bank filters

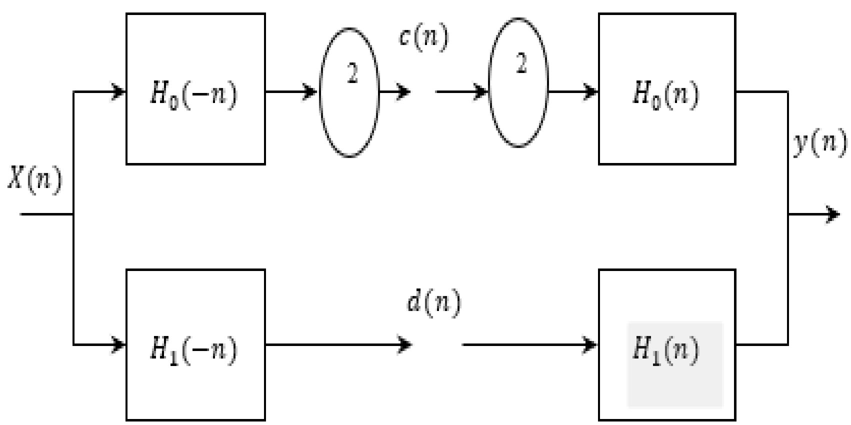

Figure 3. Block representation of wavelet transformation (single step) with reconstruction into output signal using realizable filters

Where, V_j is the single level decomposition and $W_{i,j}$ is group of V_j

The single wavelet ψ and scaling function ϕ defines the structure of a dyadic wavelet. The additional wavelet in equation 4 avoids any possibility of incompleteness. In terms of multi-resolution framework, the nesting of $V_0 \subset V_1, \; W_{1,0} \subset V_1, W_{2,0} \subset V_1$ can be demanded. Hence, the wavelets and scaling function satisfies the dilation equations

$$\begin{aligned} \varnothing(t) &= \sqrt{2}\sum_n h_0(n)\varnothing(2t-n) \\ \psi_i(t) &= \sqrt{2}\sum_n h_i(n)\varnothing(2t-n) \qquad i = 1, 2 \ldots n. \end{aligned} \tag{9}$$

$h_0(n)$ $h_1(n)$ and $h_2(n)$ from above section illustrates about wavelets $\psi_1(t), \; \psi_2(t)$ and scaling function of equation 9.

Proposed System Model

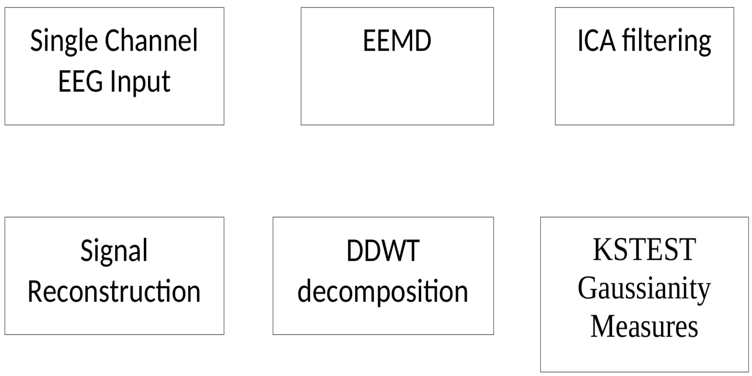

Figure 4. Proposed Architecture for EEG artifacts removal in single channel signal

The proposed architecture (figure 4) shows the functional description of prototype. Mijovic et al[7] first proposed the ICA and EEMD in combined form for separation of input signal in a given channel. The author used system to eradicate artifacts of muscle movement and eye blinks primarily from EEG signals. ECG signals were considered in later stage as the input signal and EMG as artifacts in it (separate experimentation). The proposed EEMD method segmented the input in multi-channel signal C and IMFs were generated against them (fig 5). The output of EEMG was sourced to FastICA algorithm to modify the signals according to their respective sources S (eq. 2). The ICA separates the Independent components of input IMF(s) (figure 7). The output of this step is the collective form (S) of independent components. Out of these components, a single component has Gaussian amplitude and random behavior. The KSTEST allocates '0' to this component to filter it as an artifact. The signal is reconstructed (S') again as their respective IMF(s) with minimum artifacts.

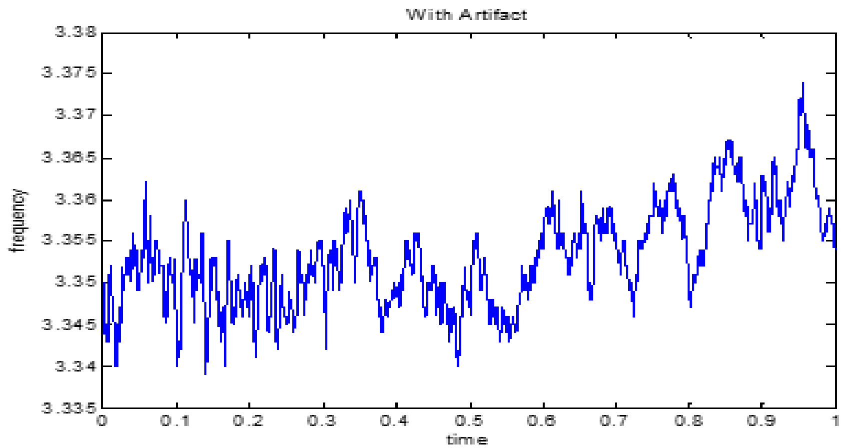

Figure 5. Input noisy EEG channel with motion artifacts for testing proposed algorithm

Before signal reconstruction, in this research an additional computation is proposed over ICA for better accuracy and higher power spectral density. The double density wavelet decomposition on IMF(s) decomposes them into a discreet level of frequencies. Figure 8 represents the wavelet coefficients for output of ICA components. Here, n components are considered as the figure is generalized notation of wavelet step. The coefficients of wavelet (for example: $cA2_1$ $cD2_1$ $cD1_1$) are further optimized for error free signal.

A filtering chain for generating wavelets through z-transform:

The wavelet coefficients for a single level (equation 7) and for multi-level (equation 8) are used to derive coefficients of ICA. The equation for multi-level decomposition will be:

$$W_{i,j} = \underset{n \epsilon Z}{Span} \left\{ \psi_i(2^j t - n) \right\} S_i, \quad i = 1, 2 \ldots n. \tag{10}$$

The coefficients are thus separate units of original signal and artifacts. To differentiate among them and discard the Gaussian components thresholding is applied. There are two common thresholding processes in this field. The first is hard-thresholding[15]:

$$\widehat{W}_{ij}(t) = \begin{cases} W_{ij}(t) & for \ |W_{ij}(t)| \geq T \\ 0 & otherwise \end{cases} \tag{11}$$

The other is soft thresholding[16] [17]:

$$\widehat{W}_{ij}(t) = \begin{cases} sgn\left(W_{ij}(t)\right)\left(|W_{ij}(t)| - T\right) & for \ |W_{ij}(t)| \geq T \\ 0 & otherwise \end{cases} \tag{12}$$

The IMF(s) reconstructed from wavelet coefficients (eq. 12) have comparatively lesser amount of artifacts compared with conventional ICA techniques and obtained from:

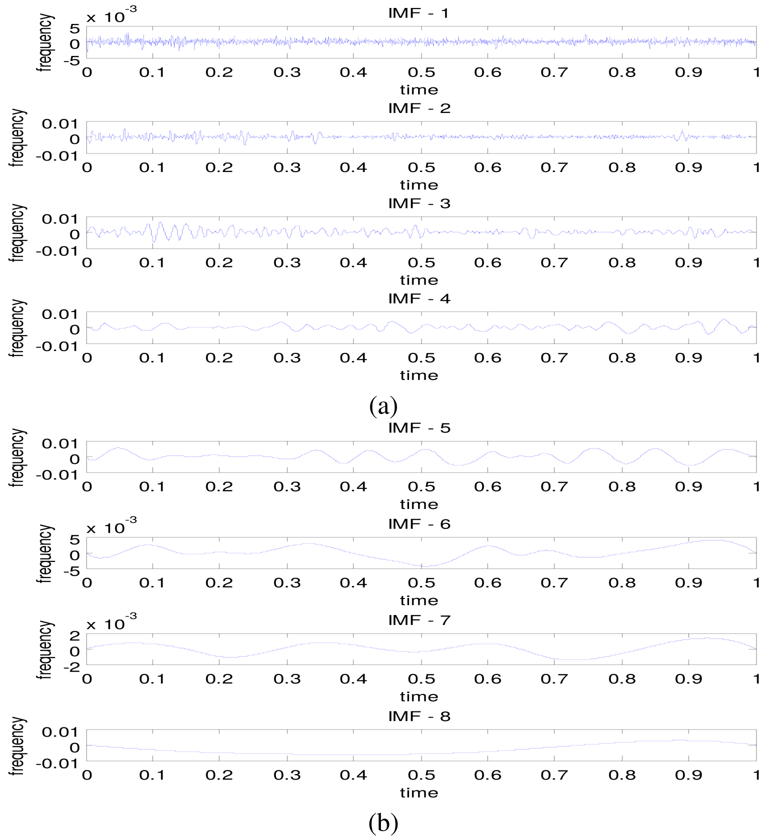

Figure 6. Output IMF(s) from EEMD. Total 8 IMF(s) are obtained for single source EEG channel

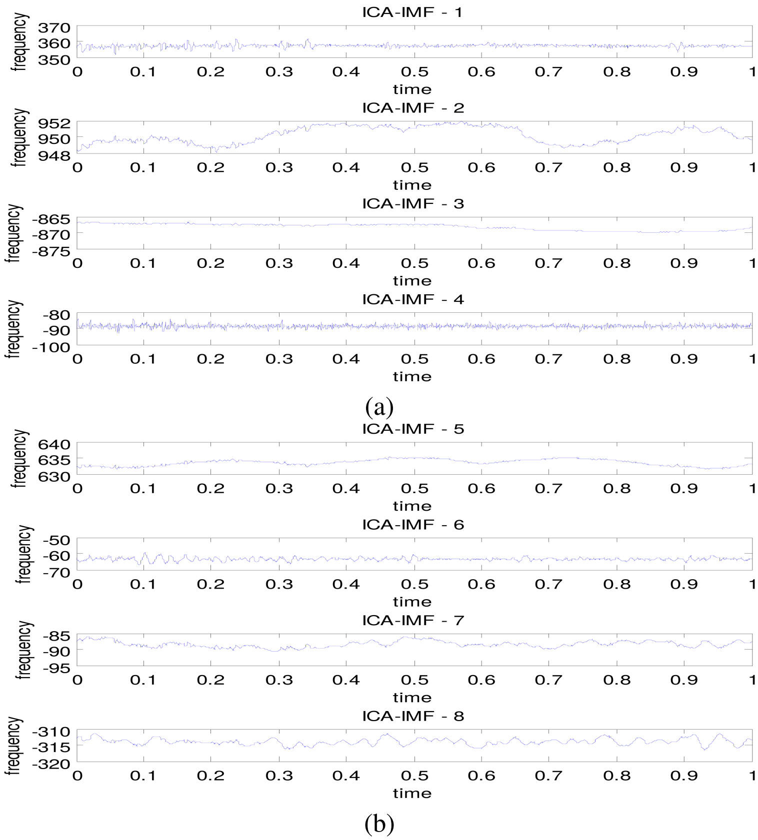

Figure 7. EEMD-ICA output components after processing for 8 outputs obtained from figure 6.

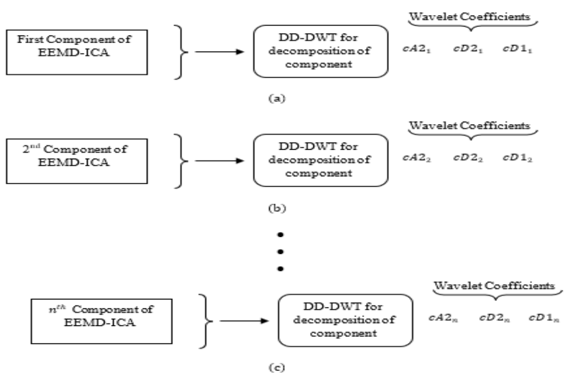

Figure 8. DWT decomposition of Single Channel Source Signal obtained from ICA

$$\widehat{S}_n = \sum_{i=1}^{k} w_{ij} \tag{13}$$

Where, $\widehat{S}_n$ is number of IMF(s) reconstructed k is unity more than level of decompositions and w_{ij} are wavelet coefficients. The original signal is thus the sum of all the m number of IMF(s) reconstructed (figure 9) from EMD:

$$y = \sum_{i=1}^{m} \widehat{S}_i \tag{14}$$

Data Acquisition

The data for experimentation is contributed by Kevin Sweeney at National University of Ireland at Maynooth. The data is available on an online open source interface[18] and description of recordings is acquired from same source only. The recorded samples are EEG samples (figure 6) contaminated by motion artifacts. Every recording is a single pair of similar psychological signals recorded in close proximity via transducers. Keeping the single transducer stationary, the second transducer is manipulated to create artifacts related to motion and of variable duration within each 2-minute interval of recording. The EEG signals and the accompanying trigger signal were digitized at 2048 Hz. Comparison of the artifact free EEG having high correlation during motion-free intervals and lower correlation during artifact-contaminated intervals defines the efficiency of proposed work.

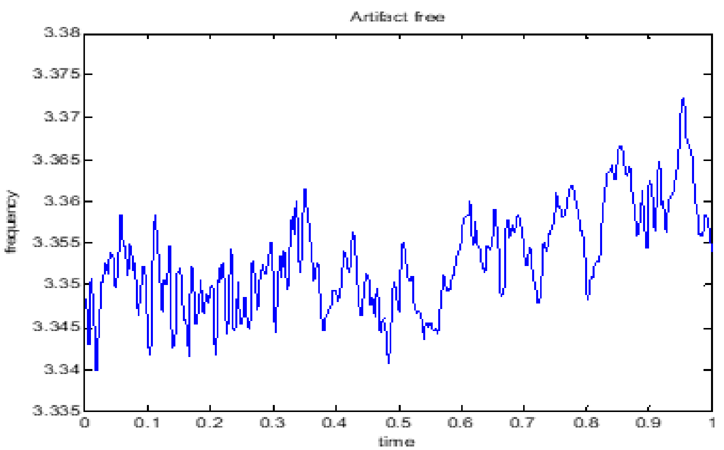

Figure 9. Artifact free EEG channel reconstructed after soft thresholding

Performance Evaluation Parameters

SNR

The SNR is a measure of low magnitude waves stimulated by some sinusoid approach. The input wave in space-time domain is studied via periodogram optimized with Kaiser Window to attenuate large side lobes. The algorithm looks for the non-zero spectral component to result fundamental frequency. The central moment of each subject is computed for all adjacent bins in a decreasing order (i.e. from maximum to minimum). These frequencies are detectable in second bin and further frequencies are the replica of these steps. The power of a signal is selected as the larger harmonic in case if the signal shows the monotonically decreasing behavior compared with neighboring signal. The function is the ratio of noise intensity in noise contaminated region derived via median power. To calculate the performance, DC component is rejected and noise at every step could be either ordinate of a point or estimated level. This noise is eliminated for an artifact free signal.

Power spectral Density

The supplemented readings of Electro encephalography signals are sourced by Gaussian noise. The obtained variation in signals in reference to theoretical model leads to restricted access in accuracy of power spectrum of signals (Table 1).

$$P_{\overline{xx}}(w) = P_{rr}(w) + \Delta P(w) \tag{15}$$

Where,
P_{rr} = Reference Power Spectrum of Artifact free signal
$P_{\overline{xx}}$ = Power Spectrum of ICA modified Signal
P = Distortion Spectrum by side effects of methods (Should be 0 ideally)

$$\Delta P_j(w) = -m_{j1}^2 P_{nn}(w) \tag{16}$$

Here,
m_{j1} Denotes weight from Matrix M
P_{nn} Component of brain signals in ICA

Equation 11 represents the minimization of ICA-EEG based on spectral function (P_{nn}) with factors m_{j1}^2. As there exists direct relationship among decrease in $j$and m_{j1}^2, front end experience high distorted signals of spectrum. DDWICA along with reduction of residual EEG signals in artifact components simultaneously minimize P_{nn} on a serious note that leads to improved approximated value of pure EEG power spectrum.

Quantifying the distortion degree that were present in readings cause of artifact minimization scheme, the mean power density to average ten epochs is calculated for conversion in decibels.

Results:

EEG signal with motion artifacts are acquired from the physionet[18]. Signals having motion artifacts have a low SNR due to the harmonics created by random frequency artifacts as shown in figure below:

The signal is decomposed to IMFs using EMD to find monotonic component of signal as a separate source for ICA decomposition. X axis is the time and y axis is the amplitude (microvolt).

A blind source separation with ICA is performed over these IMFS so that all the signals following different distribution will be separated as shown below:

By performing ks test we found that IC 4 is following the random distribution thus it has the noise component. So we are making this independent component a zero and reconstructing IMFs with

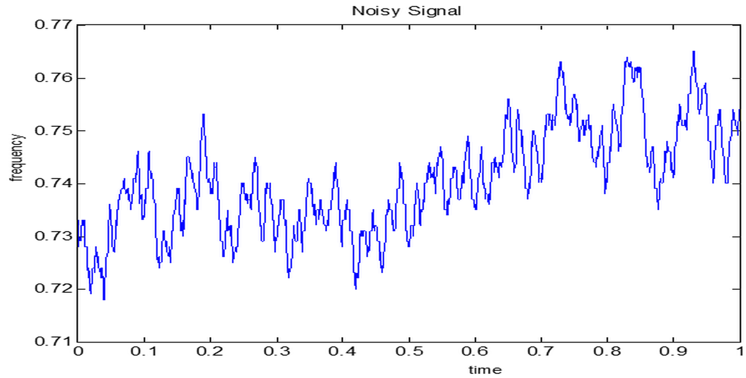

Figure 10. EEG signal with artifacts

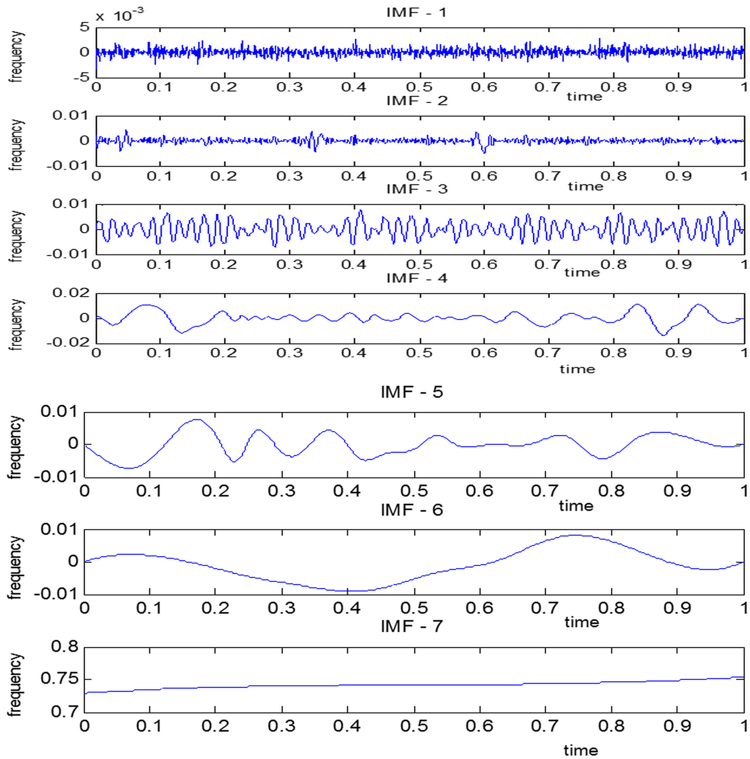

Figure 11. IMFs of EEG Signal (X axis is time and y axis is amplitude (microvolt))

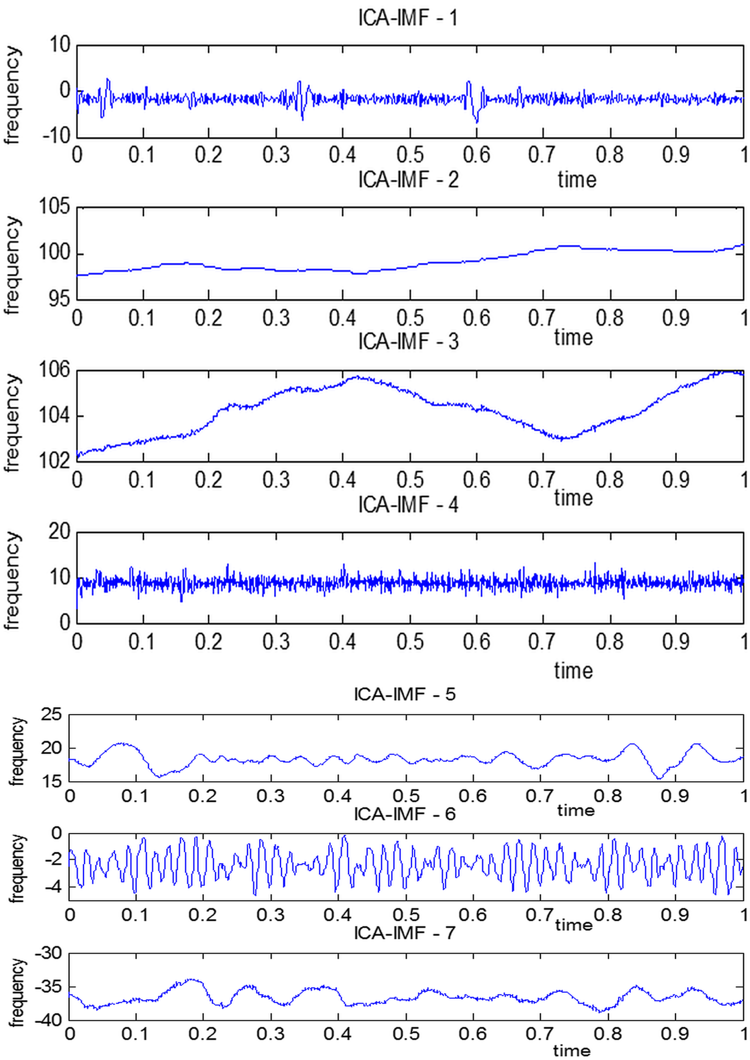

Figure 12. ICs of IMFs (X axis is time and y axis is amplitude (microvolt))

weighting matrix W. This process removed a big part of noise from the signal as the SNR for EEG signal increased by 1 dB, but to get a better version of signal we applied a 3 level DD-DWT soft denoising over the each reconstructed IMF. Reconstructed signal is as below:

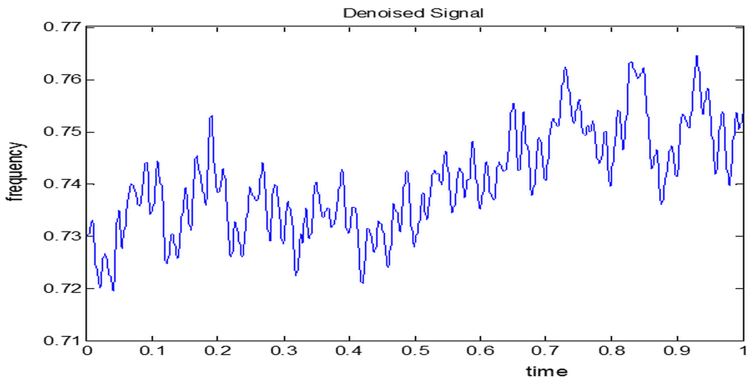

Figure 13. EEG with Removed artifacts (X axis is time and y axis is amplitude (microvolt)).

The signal quality can be evaluated over the power spectral density as show below:

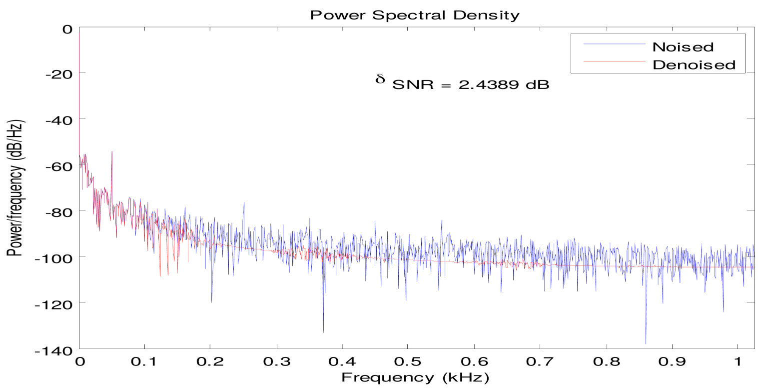

Figure 14. PSD of EEG Signals

Figure above shows the PSD for EEG with artifacts (blue) and without artifacts (Red), as observed from figure power of high frequency components has been lowered which gives a rise in SNR of 2.4389 dB.

Table below show the SNR of processed for 7 EEG signal from the two method to show the performance improvement with DD-DWT transform.

SNR values are plotted for all EEG input signal and graph shows that proposed methodology is better for EEG Artifact removal.

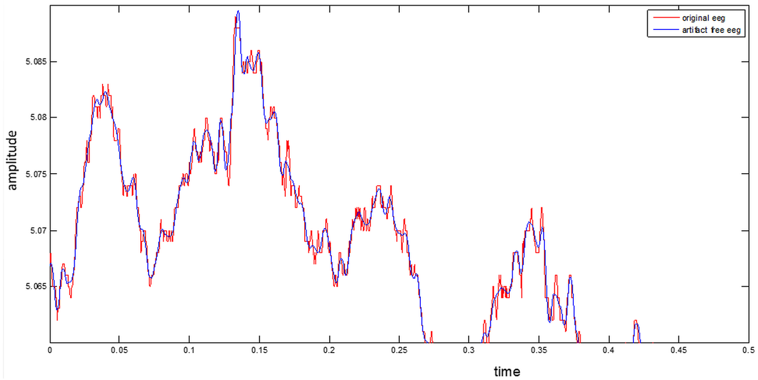

Figure 15. Noisy and Denoised signals (X axis is time and y axis is amplitude (microvolt))

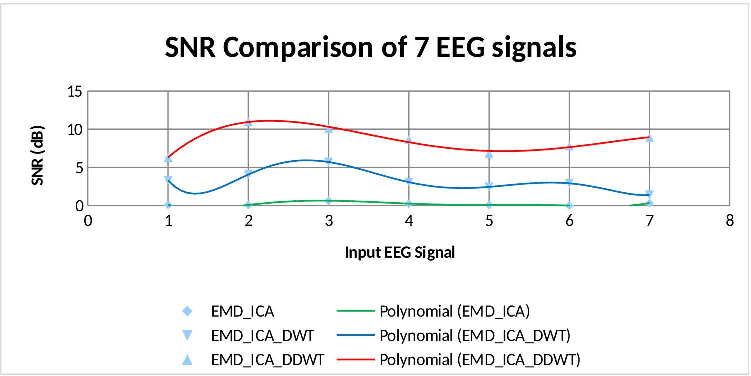

Figure 16. Comparison graph for SNR of 7-EEG Signals

Table 1. Comparative table between EMD_ICA and EMD_ICA_DWT and Proposed EMD_ICA_DDWT of PSNR for 7 EEG signals

Comparision Table for SNR of 7-EEG Signals			
Input Signal	**EMD_ICA**	**EMD_ICA_DWT**	**EMD_ICA_DDWT**
EEG1	0.0898	3.3008	6.3129
EEG2	0.0805	4.0558	11.015
EEG3	0.6359	5.6965	10.0782
EEG4	0.2529	3.0611	8.6789
EEG5	0.0849	2.4389	6.8061
EEG6	0.0022	2.9037	7.8015
EEG7	0.3315	1.3885	8.917

Conclusion

A novel technique for artifacts removal in EEG signal has been proposed. The work is a modification to the work proposed in[18]. EEG Signals has specified range of frequency with different amplitude for particular points of EEG acquisition system, but when artifacts is introduced, the range of frequency becomes higher with random high frequency components shows significant power in power spectrum. Removing these artifacts directly with filters may loss the necessary information because of amplitude variation, so we first decomposed signals to IMFs with E-EMD to convert single time samples signal to multiple sources with different frequencies. Each IMF has a small frequency range but if the signal has artifacts than IMFs will be having low amplitude of high frequency components. To separate this we performed ICA over these IMFs to get independent components of signal so that the component of artifacts will be treated as one IC with random distribution, which can easily be eliminated with the check of gaussianity and IMFs can be reconstructed from the rest ICs with inverse ICA. To get the better performance, before reconstructing Signal from IMFs we applied 3-Level double density DWT over each IMF and it coefficients are artifacts free with soft thresholding Method, these leads the signal to remove the high frequency energy components. Reconstructed signal shows much lesser power in power spectrum with average SNR rise of 2.5 dB.

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